# Hypothesis Tests

**What is a hypothesis?**

A hypothesis is a statement which might be true e.g. Sam has a hypothesis that large dogs are better at catching balls than small dogs.

A hypothesis is a prediction that is supported with an explanation. The explanation might use previous knowledge or scientific evidence, e.g. if a plant is given unlimited amount of sunlight then the plant will grow to its largest possible size; if I eat more vegetables then I will lose weight; if I brush my teeth every day then I will not need fillings.

**What is a hypothesis test?**

A hypothesis test is a statistical test to ‘prove’ a hypothesis held by a researcher. A hypothesis test compares two datasets, or a sample from a dataset. Afterwards, a conclusion will be made about the characteristics of the comparison.



**Why do we need one?**

Hypothesis testing is the process used to evaluate the strength of evidence from the sample. This is often called a research question and is basically an idea that must be put to the test. Hypothesis testing is very important in the scientific community and is necessary for advancing theories and ideas, e.g. a hypothesis test is used when you are wishing to test a new product e.g. a new drug for treatment of a disease, a claim that a new shampoo works, a new vaccine is safe, etc.

For these statistical tests, when a researcher attempts to prove some hypothesis of interest e.g. that a new treatment is more effective than the existing treatment. Initially they assume the contrary view (i.e. the new treatment is *not* more effective than the existing one) and only comes down in support of the hypothesis of interest if the data gathered is sufficiently unlikely to have been generated by the contrary view.

The contrary view is known as the **Null Hypothesis** and it is written as Ho.

What we are testing for is referred to as the **Alternative Hypothesis** and is written as H1.

**Example:** A study was conducted to compare the effect of two different pain killers on blood glucose levels. Fifteen subjects were given painkiller *A* and 12 were given painkiller *B* and the blood glucose levels recorded in mg/kg as shown in the table below. The objective of the study is to determine if the blood glucose levels are higher with one or other painkiller.

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| --- | --- |
| Painkiller A | 44, 69, 51, 71, 52, 71, 55, 76, 60, 82, 62, 91, 66, 108, 68 |
| Painkiller B | 52, 95, 64, 97, 68, 107, 77, 116, 79, 83, 84, 88 |

(a) State the Null Hypothesis of test above

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| Ho = There is no difference in blood glucose level between the two painkillers |

(b) State the Alternative Hypothesis

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| H1 = The blood glucose levels are higher in one painkiller than the other. |

For some statistical tests, once you have decided what your Null and Alternative Hypothesis are, you need to see if you have normally distributed data.

This is done by the following 3 ways

* Check that the mean and median are close
* A Histogram of each to show a bell shaped distribution
* A Shapiro-Wilks tests where the resultant p-value is >0.05 (covered later on)

For now look at the first two points.

(c) On R studio input the data and show the mean and median of Painkiller A and B along with a Histogram to show if the distribution is Normal

Write below what you found.

|  |
| --- |
| PainkillerA<-c(44,69,51,71,52,71,55,76,60,82,62,91,66,108,68)PainkillerB<-c(52,95,64,97,68,107,77,116,79,83,84,88)> summary(PainkillerA) Min. 1st Qu. Median Mean 3rd Qu. Max.  44.0 57.5 68.0 68.4 73.5 108.0 > summary(PainkillerB) Min. 1st Qu. Median Mean 3rd Qu. Max.  52.00 74.75 83.50 84.17 95.50 116.00> shapiro.test(PainkillerA) Shapiro-Wilk normality testdata: PainkillerAW = 0.9482, p-value = 0.4965> shapiro.test(PainkillerB) Shapiro-Wilk normality testdata: PainkillerBW = 0.99157, p-value = 0.9999Since the data is normally distributed we can use a t test.Note: For shaprio test.The Null Hypothesis is that your data is normal. So a value of 0.999 means we do not reject the null hypothesis and conclude the data is normally distributed. \*This is different to Hypothesis Tests. |

**P value** is the probability value is a number describing how likely it is that your data would have occurred by random chance (i.e. that the null hypothesis is true)

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The level of statistical significance is often expressed as a 𝜌−𝑣𝑎𝑙𝑢𝑒 between 0 and 1. The smaller the 𝜌−𝑣𝑎𝑙𝑢𝑒, the stronger the evidence that you should reject the null hypothesis. In simpler terms, the 𝜌−𝑣𝑎𝑙𝑢𝑒 **does not** provide any evidence in support of 𝐻0.

𝜌<0.001 indicating very strong evidence against 𝐻0.

𝜌<0.01 indicating strong evidence against 𝐻0.

𝜌<0.05 indicating moderate evidence against 𝐻0.

𝜌<0.1 indicating weak evidence or a trend.

𝜌≥0.1 indicating insufficient evidence.

A p-value of less than 0.05 is statistically significant. It indicates strong evidence against the null hypothesis as there is a 5% probability the null is correct. If the p-value is < 0.05 it means we reject the Null Hypothesis Ho and accept the Alternative Hypothesis H1.

Going back to our example about painkillers. Perform a Hypothesis test on the information above using R studio and the command *t.test(X,Y)*.

(d) State the p-value given and write what that means with relation to the context of the question.

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| > t.test(PainkillerA,PainkillerB) Welch Two Sample t-testdata: PainkillerA and PainkillerBt = -2.3382, df = 22.591, p-value = 0.02861alternative hypothesis: true difference in means is not equal to 095 percent confidence interval: -29.72983 -1.80350sample estimates:mean of x mean of y  68.40000 84.16667P value is <0.05 as it equals 0.02861, this is seen as a statistical significant result.  |

(e) What are the positive aspects of performing these Hypothesis tests, how can the information be used?

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| Since we know there is a link we can look at their averages. Painkiller A has a mean of 64 and painkiller B has a main of 84. From the hypothesis test it seems there is a difference and we see that Painkiller B results in higher blood glucose levels. This means that it may be unsuitable for certain patients, possibly those who might be diabetic. |

We will return to most of these aspects later on.

**Example 2:** In Santa Clara more than 30% of people voted in the primaries. Using Ho, H1 notation write the Null and Alternative Hypothesis.

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| x is the number of people in Santa Claran is the number of people who votedIn wordsHo = 30% or less of the population of Santa Clara voted in the primariesH1 = More than 30% of the population of Santa Clara voted in the primaries.In symbolsHo = n ≤ 0.3xH1 = n > 0.3x |

**Exercise: Null and Alternate Hypothesis – To be removed**

For the following state the null and alternative hypothesis, label them as Ho and H1.

1. Are people who live in country A heavier than those that live in country B?

$\overbar{x\_{a}}$ is the average weight of people in country A

$\overbar{x\_{b}}$ is the average weight of people in country B

Ho = People in country B are just as heavy as or heavier than those who live in country A.

H1 = People in country A are heavier than people in country B

Ho = $\overbar{x\_{b}} \geq \overbar{x\_{a}}$

H1 = $\overbar{x\_{b}} < \overbar{x\_{a}} $

2. A medical trial is conducted to test whether or not a new medicine reduces cholesterol by 25%. State the null and alternative hypotheses.

Ho = Medicine does not decrease cholesterol by 25%

H1 = Cholesterol of patients is reduced by 25%

Cb – Cholesterol before taking the medicine

CA – Cholesterol after taking the medicine

Ho = CA ≠ 0.75CB

H1 = CA = 0.75CB

3. We want to test whether the mean GPA of students in American colleges is different from 2.0 (out of 4.0).

Ho = The mean GPA for students in American Colleges is 2

H1 = The mean GPA for students in American colleges is not 2

x – the mean GPA of students in American colleges.

Ho = x = 2

H1 = x ≠ 2

4. We want to test if college students take less than five years to graduate from college, on the average.

µ– the average number of years for college students to graduate.

H0: μ ≥ 5

Ha: μ < 5

5. In an issue of U.S. News and World Report, an article on school standards stated that about half of all students in France, Germany, and Israel take advanced placement exams and a third pass. The same article stated that 6.6% of U.S. students take advanced placement exams and 4.4% pass. Test if the percentage of U.S. students who take advanced placement exams is more than 6.6%.

P – Number of pupils who take advanced placement exams.

Ho = the number of pupils who take advanced placement exams is less than or equal to 6.6%

H1 = the number of pupils who take advanced placement exams is greater than 6.6%

H0: p ≤ 0.066

Ha: p > 0.066

**Confidence Intervals**

Because the datasets we use are typically samples of a larger population, the mean that we compute for the sample is only an estimate of the population mean.

For a given sample, we can produce a range of values (an interval) in which we are fairly certain the true population mean lies.

Usually we quote these “confidence intervals” at a level of 95% certainty.

This means there is only a 5% chance that the true population mean lies outside this range.

**Example:** Using the dataset *Cholesterol\_R.csv*

Input the data into R studio

(a) Calculate the mean of *before*, write it below

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| > mean(Before)[1] 6.407778 |

(b) Calculate the mean of *After 8 Weeks*

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| > mean(After8weeks)[1] 5.778889 |

(c) Use the command *t.test*, to perform a hypothesis test and hence show the maximum and minimum difference in the two means.

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| > t.test(Before,After8weeks) Welch Two Sample t-testdata: Before and After8weekst = 1.6443, df = 33.796, p-value = 0.1094alternative hypothesis: true difference in means is not equal to 095 percent confidence interval: -0.1485269 1.4063046sample estimates:mean of x mean of y  6.407778 5.778889 |

This means that there 95% chance that the difference in the two means will be between -0.1485269 and 1.4063046

To calculate the confidence interval yourself you need the following

* Mean $\overbar{x}$
* Sample Size $n$
* Standard Deviation $sd$
* Critical value $z=1.96$ for 95%
* The data follows a normal distribution

The formula is as follows

$$CI= \overbar{x} \pm z\frac{sd}{\sqrt{n}}$$

**Example 2:** A hardware manufacturer produces bolts used to assemble various machines. Assume that the diameter of bolts produced by this manufacturer has an unknown population mean 𝝁 and the standard deviation is 0.1 mm, suppose the average diameter of a simple random sample of 50 bolts is 5.11 mm. Calculate the 95% confidence interval.

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| $$\overbar{x}=5.11 $$$z=$ 1.96$$sd= 0.1$$$n= $50Max = 5.14Min = 5.08We can be 95% sure that the true mean lies between 5.14 and 5.08 |

**Exercise: Confidence Intervals:**

1. You want to rent an unfurnished one-bedroom apartment in Boston next year. The mean monthly rent for a simple random sample of 32 apartments advertised in the local newspaper is $1,400.

Assume that the standard deviation is known to be $220.

Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

Max = 1476.23

Min = 1323.77

We can be 95% sure the true mean lies between 1323.77 and 1476.23

2. We have IQ test scores of 31 seventh-grade girls in a Midwest school district.

We have calculated that sample mean is 105.84 and the standard deviation is 14.27.

Give a 95% confidence interval for the average score in the population. What is the margin of error?

Max = 110.86

Min = 100.82

We can be 95% sure the true mean lies between \_ and \_

3. Determine the sample size needed to estimate the average weight of all second-grade boys if we want to be within 1 pound with 95% confidence.

Assume we know that the standard deviation of such weights is 3 pounds.

$$\left(1.96\right)× \frac{3}{\sqrt{n}}=1$$

N = 34.57 = 35

4. How big must the sample size be if the margin of error on the waiting times of a hospital is 2 minutes? If the standard deviation is 15 minutes and the confidence interval is 95%.

$$\left(1.96\right)× \frac{15}{\sqrt{n}}=2$$

N = 216.09 = 217