# Numerical Representation

From National 5 Applications of Mathematics you will have encountered numerical statistics before. The main numerical statistics you have encountered so far should be

Measures of average – mean, mode and median.

Measures of spread – standard deviation and SIQR.

**Recap:**

**Example 1:** Calculate the mean and median of the following test scores from Mr Williams’ class.

45 81 29 10 26 38 40 28 77 63 90

|  |
| --- |
| mean = 47.910 26 28 29 38 40 45 63 77 81 90median = 40 |

A pupil was absent from the class. What would happen to the average if the person scored 52, would the average go up or down?

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| Median would change to 42.5mean would slightly increase |

Mrs Davis’s class has a mean score of 62. Make a comparison between the two classes.

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| On average pupils did better in Mrs Davis’s class than Mr Williams’s class. |



**Example 2.** Calculate the mean and standard deviation of the following

24 62 36 48 51 37

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| mean = 43

|  |  |  |
| --- | --- | --- |
| x | $$x-\overbar{x}$$ | $$(x-\overbar{x}) ^{2}$$ |
| 24 | -19 | 361 |
| 62 | 19 | 361 |
| 36 | -7 | 49 |
| 48 | 5 | 25 |
| 51 | 8 | 64 |
| 37 | -6 | 36 |
|  | Total | 896 |

$$sd= \sqrt{\frac{896}{6-1}}=13.5$$ |

**Example 3:** Calculate the median and SIQR of the following

19 38 23 20 36 45 50 20 24 37 54

|  |
| --- |
| 19 20 20 23 24 36 37 38 45 50 54median = 36Q1 = 20Q3 = 45IQR = 45 – 20 = 25 |

**Example 4:** Look at table below showing marks obtained in a test for 88 pupils.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Marks | $$0\leq x<10$$ | $$10\leq x<20$$ | $$20\leq x<30$$ | $$30\leq x<40$$ | $$40\leq x<50$$ |
| Frequency | 6 | 16 | 24 | 25 | 17 |

Find an estimate for the mean and standard deviation

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- |
| mid(x) | f | f.x | $$(x- \overbar{x})^{2}$$ | $$f\left(x- \overbar{x} \right)^{2}$$ |
| 5 | 6 | 30 | 552.25 | 3313.5 |
| 15 | 16 | 240 | 182.25 | 2916 |
| 25 | 24 | 600 | 12.75 | 294 |
| 35 | 25 | 875 | 42.25 | 1056.25 |
| 45 | 17 | 765 | 272.25 | 4828.25 |
| Total | 88 | 2510 |  | 12408 |

$$\overbar{x}=\frac{2510}{88}=28.5 (Rounded to 1 d.p.)$$$$sd= \sqrt{\frac{12408}{88-1}}=11.9$$ |

**Exercise: Standard Deviation and Interquartile Ranges**

1. Alex records his times in minutes for a 10k race below.

 44 49 50 57

(a) Calculate the mean and standard deviation.

Mean = 50

Sd = 5.35

(b) Katrina’s times had a mean of 53 and a standard deviation of 2.3. Compare the times of Alex and Katrina.

Katrina had a slower average time than Alex but Katrina had a lower standard deviation which means her times were more consistent than Alex’s

2. Mr Thomas records the score of his class below

 88 80 76 52 64 67 70

(a) Calculate the mean and standard deviation.

Mean = 71

Sd = 11.7

(b) Mr Thomas realises there is an issue with one of the questions. He needs to increase everyone’s score by 4. How will this affect the mean and standard deviation?

Mean would change to 75

SD would be the same.

3. Freya looks at the price of her weekly shopping for the last few weeks

32 40 27 35 26 20 34 32

Calculate the median and SIQR

Median = 32

SIQR = 4

4. Graham records how long it takes for him to get to work every day.

 11 14 13 12 38 18 12 10 12

(a) Calculate the mean and median

Mean = 15.56

Median = 12

(b) Which is the better average?

The median because the mean is too high, Graham very rarely took that long to get to work.

(c) Calculate the IQR

IQR = 4.5

5. Look at the interval table below where 40 results have been recorded.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Height ($h cm$) | $$1\leq h<2$$ | $$2\leq h<3$$ | $$3\leq h<4$$ | $$4\leq h<5$$ | $$5\leq h<6$$ |
| Frequency | $$4$$ | $$6$$ | $$9$$ | $$16$$ | $$5$$ |

(a) Which interval does the median fall between?

$$4\leq h<5$$

(b) Estimate the mean

|  |  |  |
| --- | --- | --- |
| mid | f | f.x |
| 1.5 | 4 | 6 |
| 2.5 | 6 | 15 |
| 3.5 | 9 | 31.5 |
| 4.5 | 16 | 72 |
| 5.5 | 5 | 27.5 |
| Total | 40 | 152 |

$$mean=\frac{152}{40}=3.8$$

The grouped frequency table shows the length of service in years employees who have been working for a company for at least ten years.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Length of service | $$10\leq x<15$$ | $$15\leq x<20$$ | $$20\leq x<25$$ | $$25\leq x<30$$ | $$30\leq x<40$$ | $$40\leq x<50$$ |
| Frequency  | 30 | 42 | 23 | 13 | 8 | 4 |

Calculate an estimate of the standard deviation of the result.

Find the mean

|  |  |  |
| --- | --- | --- |
| x (middle) | f | f.x |
| 12.5 | 30 | 375 |
| 17.5 | 42 | 735 |
| 22.5 | 23 | 517.5 |
| 27.5 | 13 | 357.5 |
| 35 | 8 | 280 |
| 45 | 4 | 180 |
|  | 120 | 2445 |

$$̿=\frac{2445}{120}=20.375$$

|  |  |  |
| --- | --- | --- |
| $$x$$ | $$(x- ̿)^{2}$$ | $$f(x-̿)^{2}$$ |
| 12.5 | 62.01… | 1860.46 |
| 17.5 | 8.365… | 347.2 |
| 22.5 | 4.51… | 103.86 |
| 27.5 | 50.76… | 659.95 |
| 35 | 213.89… | 1711.125 |
| 45 | 606.34… | 2425.56 |
| Total |  | 7108.16… |

$$sd= \sqrt{\frac{7108.16}{120-1}}=7.73$$